SPECIFICATIONS FOR INTEROPERABILITY: FORMALIZING IMAGE SCHEMATA FOR GEOGRAPHIC SPACE

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Abstract

The formal specification of spatial objects and spatial relations is at the core of geographic data exchange and interoperability for GIS. Spatial image schemata have been suggested as highly abstract, structured schemata to capture spatial and similar physical as well as metaphorical relations between objects in the experiential world. We assume that image schemata for geographic (large-scale) space are potentially different from image schemata for table-top space. Formal definitions of four image schemata (LOCATION, PATH, REGION, and BOUNDARY) are given.

Keywords

Naive Geography, Image Schemata, Location, Region, Qualitative Spatial Relations.

1 Introduction

Exchange of data between GIS and interoperability of different vendors' GIS software are topics of enormous practical interest (e.g., documented by Buehler and McKee (1996) and the recent Interop'97 conference (1997)). Unambiguous definitions are at the core of any effort to achieve the necessary standardization that allows data exchange and co-operation of different GIS.

Standardization of technical terms and the fundamental concepts necessary to make computers interact is mostly achieved or can be achieved with current tools. The abstract behavior of computerized systems can be specified in a formal language and it requires then the checking of the compliance of the target computer system—which is by definition also a formal system—with the abstract formal system. This problem is not particular for GIS but general for all computer system standardization. The difficulties are of a practical nature and related to the lack of formal definition of most current computer languages, commercial interests in maintaining incompatible systems, and the rapid development compounded with legacy systems.

The economically important and scientifically challenging question is to describe the meaning of GIS data in terms of the real world, the so called "semantics problem." What does it mean that "P 271" is a point, "343a" a land parcel, that building "A1" is on parcel "343a," A-town is on the B-river etc., and how is this meaning communicated between systems. The naive assumption that a "rose is a rose is a rose" (Gertrude Stein) is obviously not correct: the definitions of simple geographic properties differ from country to country, despite corresponding names (Chevallier 1981; Mark 1993; Kuhn 1994).

Image schemata describe high level, abstract structures of common situations, most of them expressing spatial relations (Johnson 1987). Image schemata (Johnson 1987; Lakoff 1987) are the fundamental experiental elements from which spatial meaning is constructed, but so far image schemata have resisted formal descriptions. This paper formalizes a number of image schemata important in the geographic context (LOCATION, PATH, REGION, and BOUNDARY). This investigation is, therefore, part of the quest for naive or commonsense physics (Hayes 1978; Hayes 1985; Hobbs and Moore 1985) and "Naive Geography" (Egenhofer and Mark 1995).

The next section argues why the formalization of spatial relations in geographic space is crucial for further advances in the standardization and interoperability of GIS. Despite large strides in some small specialized areas—in particular topological relations—not much progress has been made in general. The program to formalize spatial image schemata as conceived by humans has not been completed yet. In Section 3 the specification of image schemata is discussed and Section 4 gives a comprehensive method to discover and formally describe image schemata. Section 5 explains the geographic space image schemata (i.e., LOCATION, PATH, REGION, and BOUNDARY) considered and presents their formalizations.

2 Formalizing Spatial Meaning

The spatial domain—in which GIS facts are situated—is fundamental for human living and one of the major sources for human experience (Barrow 1992). Human language exploits the communality of spatial experience among people and uses spatial situations metaphorically to structure purely abstract situations in order to communicate them (Lakoff and Johnson 1980; Johnson 1987). The formalization of spatial relations has, therefore, been an active area of research at least since 1989 (Mark *et al.* 1995).

Topological relations between simply connected regions were treated in (Egenhofer 1989) and extensive work has followed from this (Egenhofer 1994). Metric relations between point-like objects, especially cardinal directions (Frank 1991a; Frank 1991b; Freksa 1991; Hernandez 1991) and approximate distances (Frank 1992; Hernandez *et al.* 1995; Frank 1996b) were discussed. Other efforts dealt with orderings among configurations of points (Schlieder 1995) and formal descriptions of terrain and relations in terrain (Frank *et al.* 1986), but formal methods were also used to formally describe the working of administrative systems (e.g., cadastre (Frank 1996a)). Linguists have made systematic efforts to clarify the meaning of spatial prepositions (Herskovits 1986; Lakoff 1987). However, it remains an open question how to combine these interesting results within a uniform system and to apply them systematically to other examples.

The specification of spatial relations is of great practical interest to define spatial relations in spatial query languages unambiguously; the current plethora of proposals for spatial relations to complete database query languages is useless unless the relations are formally specified (which is the case for the standard relations in SQL) (Egenhofer 1992). The formal properties are the base for query optimization. Image schemata are considered good candidates as a foundation for the formal definition of spatial relations. Kuhn has pointed out the importance of image schemata as a tool to build "natural" (i.e., cognitively sound) user interfaces for GIS (Kuhn and Frank 1991; Kuhn 1993).

3 Specification of Image Schemata

Johnson (1987) proposes that people use recurring, imaginative patterns—so-called *image schemata*—to comprehend and structure their experiences while moving through and interacting with their environment. Image schemata are supposed to be pervasive, well-defined, and full of sufficient internal structure to constrain people's understanding and reasoning. They are more abstract than mental pictures and less abstract than logical structures because they are constantly operating in people's minds while people are experiencing the world (Kuhn and Frank 1991). An image schema can, therefore, be seen as a very generic, maybe universal, and abstract structure that helps people to establish a connection between different experiences that have this same recurring structure in common. Table 1 gives a selective list of Johnson's (1987 p.126) image schemata.

Container	Balance	Compulsion
Blockage	Counterforce	Restraint Removal
Enablement	Attraction	Mass-Count
Path	Link	Center-Periphery
Cycle	Near-Far	Scale
Part-Whole	Merging	Splitting
Full-Empty	Matching	Superimposition
Iteration	Contact	Process
Surface	Object	Collection

Table 1: Selective list of image schemata (Johnson 1987 p.126).

3.1 Previous Formal Description of Image Schemata

Despite efforts, success in specifying spatial image schemata has been limited. An early paper (Kuhn and Frank 1991) gave algebraic definitions for the CONTAINER ("in") and SURFACE ("on") schemata for a discussion of user interface design. At the level of detail and for the purpose of the paper, the two specifications were isomorphic (as pointed out in Kuhn and Frank (1991)). A recent effort by Rodríguez and Egenhofer (1997) introduced more operations and differentiated the CONTAINER schema from the SURFACE schema for small-scale space, using operations such as *remove*, *jerk*, and *has_contact*, and compared the application to objects in small-scale space and in large-scale (geographic) space.

In a recent paper (Frank 1998) formal descriptions for the small-scale space image schemata CONTAINER, SURFACE, and LINK were given (corresponding to the German prepositions "in", "an", and "auf") and some of the methodological difficulties reviewed.

3.2 Definition of the Concept of an Image Schema

The concept of image schemata is not well defined in the cognitive and linguistic literature (Lakoff and Johnson 1980; Johnson 1987; Lakoff 1987). Researchers in the past have used a working definition that implied that image schemata describe spatial (and similar physical) relations between objects. Most have concentrated on spatial prepositions like "in", "above", etc. and assumed that these relate directly to the image schemata (Freundschuh and Sharma 1996).

Image schemata are seen as fundamental and independent of the type of space and spatial experience. But a single schema can appear in multiple, closely related situations. For example, "in" is used for a bowl of fruit ("Der Apfel ist in der

Schale."—"The apple is in the fruit bowl."), but also for closed containers ("Das Geld ist im Beutel."—"The money is in the purse."). “Prototype effects” as described by Rosch (Rosch 1973a; Rosch 1973b; Rosch 1978) also seem to apply. For example, a different level of detail can be selected to describe the same image schema.

3.3 Language Dependence of Particular Image Schemata

It is possible that image schemata provide language-independent building blocks for structure and different languages may combine the building blocks differently; the list of image schemata overlaps with Wierzbicka’s list of universal language primes (Wierzbicka 1996). Formally, the topological relations described by Egenhofer have been used in this way to define more complex topological relations (Mark *et al.* 1995). The obvious differences between languages are one important point in the cultural difference that limits the use of GIS (Campari and Frank 1995) and the problem is further aggravated by regional differences within a language.

3.4 Methods to Formalize Image Schemata

3.4.1 Predicate Calculus

Lakoff (1987) gives a definition of the CONTAINER schema using predicate calculus. In theory, predicate calculus has all the expressive power necessary, but it is practically limited by the frame problem, which makes succinct definition for changes impossible (Hayes 1977; McCarthy 1985)

3.4.2 Relations Calculus

The behavior of topological relations (Egenhofer 1994; Papadias and Sellis 1994), but also cardinal directions and approximate distances (Frank 1992; Frank 1996b) can be analyzed using the relations calculus (Schroeder 1895; Maddux 1991). Properties of relations are described as the outcome of the combination (the ";" operator) of two relations. The description abstracts away the individuals related (in comparison to the predicate calculus) and gives a simple algebra over relations. This leads to succinct and easy-to-read tables, as long as the combination of only few relations is considered.

a (R;S) c = aRb and bSc
for example: North;NorthEast = {North or NorthEast}
meet;inside = {inside, covered, overlap}

3.4.3 Functions

Functions are more appropriate to capture the semantics of image schemata with respect to operations,. Relation composition is replaced by function composition (the "." operator). In order to use this notation flexibly, a “curried” form of function writing must be used (Bird and Wadler 1988; Bird and Moor 1997).

$f \cdot g\,(x) = f\,(g\,(x)).$

Function composition can be described by tables as well, but these grow even faster than relation composition tables. Axiomatic description as algebra are more compact but also more difficult to read.

3.4.4 Model Based

A model of the scene is constructed and used for reasoning (there is some evidence that this is also one of the methods humans apply (Knauff *et al.* 1995)). A fundamental set of operations to construct any possible state of this model and a sufficient number of observe operations to differentiate any of these states are provided. In addition, more complex operations can be constructed using the given operations.

The simplest model is to use the constructors of the scene directly and to represent each scene as the sequence of constructors which created it (Rodríguez 1997). This gives a (possibly executable) model for functional or relation oriented description.

Models can be ontological—modeling some subset of the existing world—or they can be epistemological—modeling exclusively the human conceptualization of the world. From an ontological model, more than one epistemological view can follow.

3.5 Tools Used

Formal specifications written and checked only by human minds must be regarded with great skepticism: humans are not particularly apt in finding errors in formal descriptions. For effective work, formal (computerized) tools must be used. Two types have been used: Logic based languages (e.g., Prolog (Clocksin and Mellish 1981)), used for the definition of spatial terminology (Frank *et al.* 1986) and for spatial relation calculus (Egenhofer 1989). Logic based systems must use “extralogical” operations when change is considered (*assert* and *retract* in Prolog). Recently, functional languages (Bird and Wadler 1988) have been advocated (Frank 1994; Frank 1996; Kuhn and Frank 1997), especially Haskell (Peterson *et al.* 1997) and Gofer (Jones 1991; Jones 1994). Allegories (a special kind of categories) provide the theoretical structure to unify the two approaches (Bird and Moor 1997).

4 Method to Discover and Describe Image Schemata

In (Frank 1998) the method used by linguists (Lakoff 1987; Lakoff and Johnson 1980) is applied to formalization studies: Natural language sentences are given which describe a common spatial situation and suggest an interpretation or logical derivation, which is not directly expressed. The logically implied and tacitly deducible conclusions from a description—most often centered around the assertion of a spatial preposition (e.g., in)—are taken as the contents of the image schemata, i.e., the abstract structure expressed with it. A number of restrictions and assumptions are necessary to make progress with this investigation:

4.1.1 Operational Definition of Image Schemata

As an operational definition of image schemata we consider spatial situations image schemata if they are usable as a source domain for metaphorical transfer to some target domain; this demonstrates that a commonly understood structural content, that is independent of the specific situation, exists.

4.1.2 Assumption of Polysemy

A single word may have multiple meanings (e.g., the English word “spring” can be the verb “to jump”, a season, a source, etc.). We assume that polysemy helps to initially separate what are potentially different meanings of a word for formalization (Johnson 1987). If the meanings are the same after formal description is achieved, the assumed polysemy can be dropped. In particular, we assume here that spatial prepositions are polysemous with the context, i.e., “in” for small-scale (table-top) space as investigated in (Rodríguez and Egenhofer 1997; Frank 1998) and “in” for geographic space, as investigated here, are assumed to be two homonyms.

4.1.3 Exclusion of Partial Spatial Relations

Spatial relations may be partial: a pen may be partially on a sheet of paper, a city boundary partially in one, partially in another state or country (e.g., Niagara Falls is a city both in Canada and the U.S.A.). At the present time such situations are excluded from consideration and their analysis is postponed. Ongoing work by Egenhofer to differentiate situations with the same topology by metric measures characterizing the degree of overlap etc. may become useful.

4.1.4 Restriction to a Single Level of Detail and Abstraction

The level of abstraction differs depending on the requirements of the situation (Timpf *et al.* 1992; Voisard and Schweppe 1994; Voisard and Schweppe 1997). These multiple levels of detail play an especially important part in geographic space and make the specification of image schemata difficult. Level of detail may be spatial subdivision, may be more rule considered or may be the subdivision of categories into subcategories (Jordan *et al.* 1998; Giunchiglia and Walsh 1992). All these effects are excluded from this investigation.

4.1.5 Concentration on Specific Environment, Here Geographic Space

We assume that image schemata for geographic space are separate from image schemata for small-scale space (Montello 1993; Couclelis 1992). Some suggested image schemata in (Johnson 1987) use terminology from geographic space (e.g., PATH), others suggest that the same image schema is used for different types of spaces. If the same terminology is used, we assume here—for methodological reasons—polysemy (i.e., the same word is used with different meanings).

4.1.6 Concentration on a Single Language and Epistemology

The examples given here are in German (with English translations) as this is the authors' native language; the results can be compared with the English language situation and some differences observed (Herskovits 1986; Montello 1995). The language examples are the driving force here and the concentration is on the epistemology—no attempt to achieve an ontology is made.

5 Geographic Space Image Schemata

The subset of reality considered here consists of some geographic-space-objects plus the immediate relations between them. The geographic-space-objects are of the following types:

- LOCATION: This image schema is missing in Johnson's list but seems to be important for geographic space. We use it as a precise position in space.
- PATH: A PATH connects places and consists of a starting point and an endpoint and points in-between these two, as given in (Johnson 1987).
- REGION: We use this image schema similar to Johnson's CONTAINER schema. A REGION has an inside and an outside.
- BOUNDARY: This image schema is similar to Johnson's CENTER-PERIPHERY schema, but BOUNDARY is also part of Johnson's CONTAINER schema description.

These objects cannot move and their relations are fixed (but not precisely known). The immediate relations are relations that exist without the interference of another object.

In addition, movable objects such as PERSONs and their location in space are considered.

The concrete examples are taken from the Eastern European environment (Figure 1).

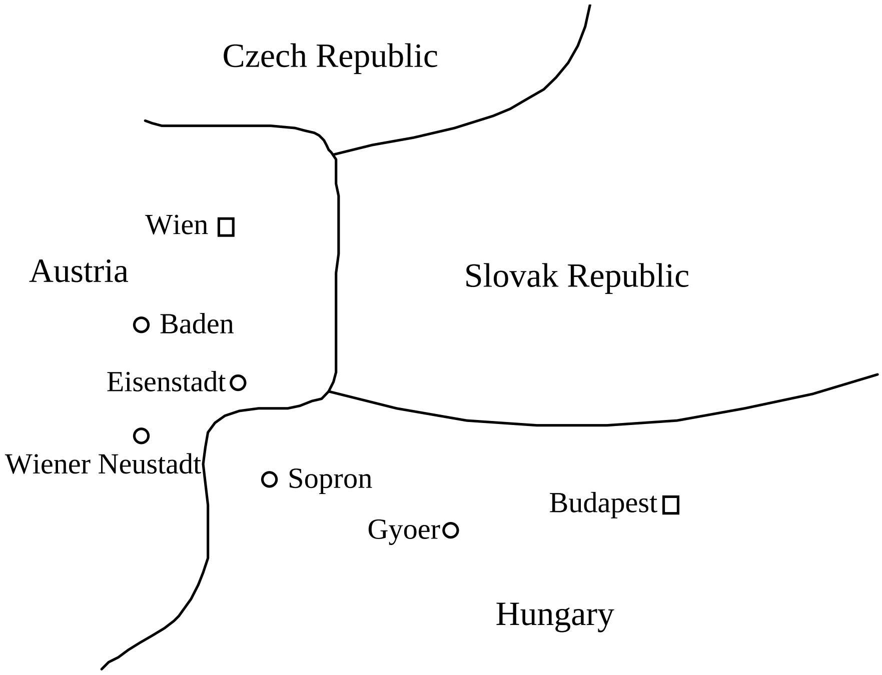

Figure 1: Map of Eastern European Environment.

The subset of reality considered is similar but not the same as discussed by Lynch (1960) who lists NODE, EDGE, REGION, PATH, and LANDMARK. Lynch was interested in the description of city form and not large-scale geographic space and the set of objects and relations considered are, therefore, different.

We first treat the relations between the geographic objects and then the movement of persons between them. The relations among geographic objects are static and can, therefore, be formalized with predicate calculus. For each relation given, a converse relation exists. Relations are written in a prefix notation (similar to a predicate). Path (a,b) means there exists a path from a to b.

This world is closed in the logical sense (Reiter 1984): everything is known about the scene and what is not known can be assumed to be false. In particular, there are no unknown objects, all objects have different names and all relations are known or inferred from the image schemata.

5.1 Base Relations

A scene is represented by a number of facts, which seem to be cognitively salient and basic and are without redundancy. In particular, we prefer relations that are simple (i.e., which are partial functions). There is no cognitive justification for these choices of base relations—other relations could be selected. For the scenes considered, we use two simple relations, i.e.,

- location in region, and
- region inside region

and two non-simple relations which are symmetric, i.e.,

- location directly connected to location, and

- region borders region.

For each base relation, a function with two parameters to test the existence of a particular fact and a function with one parameter to return a list of the related values are constructed. Finally, there is a relation indicating where a person is.

5.2 Location and Relations between Places

A path connects places. We differentiate between the simple “direct path” and the “indirect path”, which consists of a sequence of “direct paths.” At this level, different types of paths are not differentiated (i.e., no particulars of railways, highways, etc. are considered).

5.2.1 Direct Path

Connects places directly, without any intervening place (at the level of detail considered). A direct path has a start and an end location. There is, at this level of detail, no need to model path as an object, just as a relation between two places.

Es gibt einen Weg von Wien nach Baden.

There is a way from Vienna to Baden.

For this environment (but not for a city with one-way streets) the path relation is symmetric:

a “direct path” b => b “direct path” a

Path is its own converse relation:

conv (path (a, b)) = path (b, a) = path (a, b)

It is derived from a non-redundant base relation as the symmetric completion.

Du kannst von Baden nach Wien fahren und am Abend wieder zurueck.

You can drive from Baden to Vienna, and back in the evening.

5.2.2 Indirect Path

An indirect (transitive) path connects two locations through a sequence of direct path relations, such that the end location of one direct path is the start location of the next path.

gpath (a,b) = [path (a, a1) & path (a1, a2) & path (a2, ...) & ... & path (..., bn) & path (bn, b)]

conv (indirect path) = indirect path

The generalized path is derived using transitive closure. The details of the algorithm are particular to deal with cyclic and bi-directional graphs, well known as shortest path algorithm (Dijkstra 1959; Sedgewick 1983).

5.2.3 General Connection : “ueber” or “durch”

Wenn du von Wien nach Budapest faehrst, dann faehrst du durch Gyoer. Der Weg von Graz nach Wien fuehrt ueber Baden und Wiener Neustadt.

If you drive from Vienna to Budapest, you will drive through Gyoer. The way from Graz to Vienna goes through Baden and Wiener Neustadt.

A generalized path goes “via” its intermediate locations:

path (a to b via c) => path (a, c) & path (c, b)

5.2.4 Detour

A path has a length and generally there are several paths between two locations, some of them shorter than others. The concept of an “Umweg” (detour) is a path that is longer than the shortest path.

Der Weg von Wien ueber Sopron nach Budapest ist ein Umweg. Der direkte Weg fuehrt ueber Gyoer.

The way from Vienna to Budapest through Sopron is a detour. The direct route goes through Gyoer.

A path x from a to b is a detour if

detour (path (a to b via c)) if length (path (a to b via d)) < length (path (a to b via c))

5.3 Relations with Region

5.3.1 Region inside Region

A region can be inside another region (asymmetry) (Soja 1971).

Die Steiermark ist in Oesterreich.

Styria is in Austria.

The converse of inside is contains.

5.3.2 Indirect inside

Inside for region is transitive: if region1 is in region2 and region2 is in region3, then region1 is indirectly in region3.

in (region1, region3) <=> in (region1, region2) & in (region2, region3)*

Die Steiermark ist in der EU.

Styria is in the EU.—(because Austria is in the EU)

Indirect inside is the transitive closure for inside. Indirect contains is the converse.

5.3.3 Location Within Region

Wien ist in Oesterreich. Graz ist in der Steiermark. Budapest ist in Ungarn.

Vienna is in Austria. Graz is in Styria. Budapest is in Hungary.

If something is within a region and this region is within another region, then the thing is in the enclosing region as well (transitivity of the “in region” relation).

Graz ist in Oesterreich.

Graz is in Austria.

A location can be indirectly in a region: if the location is in a region1 and this region1 is indirectly in region2 then the location is indirectly in region2:

in (loc1, region2) <=> in (loc1, region 1) & in* (region1, region 2)*

The converse is the contains relation for region to all contained locations.

5.4 Relations with Boundaries

Regions have boundaries, which can be conceived as determined, sharp lines, or one of the different types of indetermined boundaries (Burrough 1996; Burrough and Frank 1996; Smith 1995)

5.4.1 Neighbor

Ungarn grenzt an Oesterreich und die Slowakei.

Hungary borders upon Austria and the Slovak Republic.

(implies that Austria borders Hungary)

Neighbor is a non-simple (a region can have several neighbors) but symmetric relation. The converse of neighbor is the neighbor relation itself. It is constructed from the non-redundant known relation.

5.4.2 Island

Das Land Wien ist vollstaendig von Niederoesterreich umgeben. Grossbritannien ist eine Insel.

The territory of Vienna is completely surrounded by Lower Austria. Great Britain is an island.

A region is surrounded by another region (i.e., is an island) if it has only one neighbor.

5.4.3 A Path Crosses a Boundary: Ueberqueren

If a path leads from a location in one region to a location in another region, it passes a boundary:

Wenn du von Wien nach Budapest faehrst, musst du die Grenze in der Naehe von Gyoer passieren.

If you drive from Vienna to Budapest, you will have to cross the border near Gyoer.

Die Strasse von Graz nach Udine passiert die Grenze bei Tarvisio.

The road from Graz to Udine crosses the border at Tarvisio.

The converse of crosses is crossed-by. The relation extends to indirect path.

The issue of the level of boundary—related to the level of the region (county, district, country etc.)—is not considered here.

This is an application of Jordan's curve theorem: "A simple closed curve (i.e., the topological image of a circle) lying in the plane divides the plane into precisely two regions and forms their common boundary." (Alexandroff 1961 p.2). Boundaries and paths are duals, but this duality is cognitively not salient (the dual of the locations are the points where several boundaries meet).

A path crosses a boundary if its start and end point are not in the same region:

crossesBoundary (a, b) = not (inSameRegion (a, b))
inSameRegion (a, b) = in (a, r) & in (b, r)

5.4.4 Boundary Towns

A location is a boundary location if there is a direct path to a location in another region:

Sopron liegt an der Grenze.

Sopron is at the border.

OnBoundary (loc a) => exist directPath (loc a, loc b) & notInSameRegion (loc a, loc b)

5.4.5 Boundary between Locations (zwischen)

A boundary is between two locations if the direct (or indirect) path from one to the other crosses the boundary:

Die Grenze zwischen Ungarn und Oesterreich liegt zwischen Eisenstadt und Sopron.

The border between Hungary and Austria is between Eisenstadt and Sopron.

… zwischen Eisenstadt und Sopron.

... between Eisenstadt and Sopron.

between (loc a, loc b) => exist directPath (loc a, loc b) & notInSameRegion (loc a, loc b)

5.5 Persons (and Other Autonomous and Movable Objects)

5.5.1 *in*

Persons are ‘in’ places and remain there unless they move.

Peter ist in Graz. Max ist in der Steiermark, er kann nicht in einem Café in Wien sitzen!

Peter is in Graz. Max is in Styria, he cannot sit in a coffee house in Vienna!

They can only be ‘in’ one place at a time. The relation is a function from person to location (for each person there is exactly one location); the location may not be known and, therefore, the relation is partial. The converse relation is “who is ‘in’?”

5.5.2 *move*

Persons move to places and are then ‘in’ the place, unless they move further:

Er ist nach Gyoer gefahren, jetzt wartet er dort auf dich.

He went to Gyoer, now he is waiting there for you.

scene2 = move (place1, scene1) => isIn (place1, scene2)

If a person is found ‘in’ place p1 at time t1 and place p2 at time t2 one can deduce a move:

Simon war letzte Woche in der Steiermark, jetzt ist er wieder in Wien.—Ist er am Samstag oder am Sonntag nach Hause gefahren?

Last week Simon was in Styria, now he is back in Vienna.—Did he drive home on Saturday or Sunday?—(move inferred in the time in-between)

5.5.3 Persons in Regions

A person can be at an unspecified location within a region:

Er ist in Ungarn auf Urlaub.

He is on vacation in Hungary.

5.5.4 Deduce “in” Region from “in” Location

If a person is at a location and the location is inside a region, then the person is in the region:

Er ist in Budapest, daher ist er auch in Ungarn.

He is in Budapest, therefore, he’s also in Hungary.

‘in’ (X, loc a) & in (loc a, region) => in (X, region)

If a person is on a path and the path is in a region, then the person is in the region:

Simon ist in Oesterreich, er ist auf dem Weg von Graz nach Wien.

Simon is in Austria, he is on the way from Graz to Vienna.

5.5.5 *Conditions for Move*

To move requires for a person some preconditions, unestablishes (retracts) some facts, and establishes new facts:

move (p, a, b): in (p, a) & path (a, b)
unestablish (in (p, a)), establish (in (p, b))

A person cannot move from one place to another unless there is a path:

Du kannst von Baden nicht direkt nach Schwechat fahren, du musst ueber Wien fahren.

You cannot drive directly from Baden to Schwechat, you have to go through Vienna.

If the person is at an unspecified location within a region, then it is only required that there is a path from every location in this region to the target.

5.5.6 *Position on Path*

Er ist auf dem Weg zu dir. Er ist zwischen Wien und Salzburg.

He is on the way to you. He is between Vienna and Salzburg.

onTheWay (X, a, b) ==> at (X, a, scene 1), path (a, b, scene1), between (X, a, b, scene2), at (X, b, scene3),
arrives (X, b) ==> previousOnTheWay (X, ?, b)

This is a hierarchical decomposition of the single move in two steps, to leave and to arrive—it is not further considered here.

5.6 Checks for Inconsistencies

The set of base relations contains minimal redundancy induced through the rules of the image schemata. Nevertheless, inconsistencies can be introduced:

- A person cannot be at a location in region a and in region b;
- in a region a and on a path that is not (at least partially) inside a.

In a formal model, guards against the introduction of such inconsistencies can be built in; this is known in the database literature as consistency constraints.

5.7 Formal Executable Model

A formal, executable model for the relations presented here has been written in a functional programming language. If a suitable set of support operations to deal with relations is available, the content of the image schemata is expressed in about 60 lines of code.

The difficulties of coding have mostly to do with finding consistent conventions to name all the relations. If the operations are written “curried”, then most rules can be written as equations between relations and relation transforming functions (i.e., point-free in the categorical sense (Bird and Moor 1997)), and in nearly all the scenes the last argument can be dropped, indicating the formulae are valid for any scene.

The use of a typed relation calculus with polymorphism allows to overload relation names; for example “a location in a region” and “a region in a region” can be reduced to a polymorphic in:: a -> b -> Bool (with two type variables a and b) and instantiations in: Location -> Region -> Bool and in: Region -> Region -> Bool. This is not only “syntactic sugar”, but forces a restructuring of code following the types of the objects related and leads to the identification of commonality. If this code is integrated with the code for relations in table-top (small-scale) space, then the assumption of polysemy can be given up if not justified.

6 Open Questions

6.1 Methodological

The method used here is borrowed from linguistics. For linguistic demonstrations, a single utterance which is acceptable by a native speaker is sufficient to demonstrate the existence of a construct. Is a single commonsense reasoning chain as given here sufficient? It documents that at least a situation exists where the suggested spatial inference is made—thus it demonstrates at least one aspect of a spatial relation in (one human's) cognition. In order to verify the universality of such spatial inference mechanisms, extended human subjects testing among people with different native languages is needed.

6.2 Language Independent Primitives

Can language independent primitives be identified (in the sense of Wierzbicka (1996))? Investigation of the same domain by researchers with different mother tongues would be necessary (or at least a collection of the related natural language descriptions). For the domain and examples here, the spatial inferences are also correct in the translations, but the use of spatial prepositions differ between German and English.

6.3 Relation between Relations and Functions

The use of category theory to establish a common theoretical ground for a relation (static) view and a function (dynamic) view is new and must be further explored. A category can be constructed over both functions and relations (Bird and Moor 1997). It is also possible to map relations into functions (aRb --> f (a,b) : Bool) and functions into a relation (f (a):: b --> aRb). Certain formalizations seem to be easier in the one, others in the other.

6.4 Composition and Interaction of Image Schemata

The combination of multiple image schemata and the interaction of image schemata with object's properties must be further explored. For an object to move along a path, it must be of the appropriate kind (only trains run along railway lines, cars cannot follow a foot path, etc., and similar restrictions apply in other cases). Possibly, the current approach trying to capture image schemata with the definition of spatial prepositions is too limited. Raubal *et al.* (1997) used prepositions and semantic connotation to investigate superimpositions of image schemata. Another interesting approach is to look at affordances. Affordances seem to be closely related to image schemata because both of these concepts help people to understand a spatial situation in order to know what to do (Gibson 1979). Affordances might be operational building blocks of image schemata but further research in this area is needed (Jordan *et al.* 1998).

Type theory as used in today's advanced programming languages (Jones 1994) provides a flexible framework that could capture the category structure of subcategories and their interaction with image schemata, but further work is necessary.

6.5 Comparison with the Modeling of Other Domains and Integration of Image Schemata Across Domains

This model of geographic space image schemata must be completed with other models, e.g., the environment of a journey (path, roads, junctions, etc.) or a city scape (Lynch 1960). If these image schemata are formally described and the interaction between image schemata and the category structure is clear, integrated models can be achieved, parallels identified, and duplication removed.

6.6 Are Image Schemata the Smallest Constituent Parts of Spatial Cognition?

Are image schemata the atoms of spatial cognition or are there smaller semantic units from which image schemata can be composed? It appears as if these were smaller pieces from which the more complex image schemata could be built, but one could also argue that these are the image schemata proper.

7 Conclusions

This very restricted set of objects from geographic space leads to a rich set of relations between them. From 5 base relations around 15 meaningful relations (not counting the corresponding converse relations) can be deduced. The commonsense knowledge of this environment is captured in a strong set of implications following from individual relations. It may be surprising how much deduction is actually possible at this high level of abstraction, where neither form nor location of individual objects are considered.

This domain is very powerful as a source of metaphors. For each of the concrete usages given here a corresponding metaphorical usage can be suggested (Lakoff and Johnson 1980; Lakoff 1987; Johnson 1987). Geographic space is typically used to structure the space of ideas—one could posit an overarching metaphor “the world of ideas is like geographic space”: ideas are connected (by logical paths), people have arrived at some position, but not yet moved on to a new understanding, in order to move from one camp (political party) to another, one has to cross a boundary… This corresponds well to the “life is a journey” metaphor (Lakoff 1987; Johnson 1987) where the journey is used to structure a large number of aspects of our understanding of our lives.

The investigation has pointed out that in this domain most of the relations are static and geographic objects do not move, only people move among them (a key concept in the definition of geographic space). This lets us at least conjecture that geographic space is selected as a source domain for the metaphorical discussion of ideas, because ideas are seen as unmovable, only the position people hold can change, not the ideas themselves (this may not be accurate truth, but is the conceptualization of ideas).

Formal descriptions of spatial relations as they are encountered in everyday life are very important for GIS. They can be used to formally define query language predicates and to optimize the execution of spatial queries. They are crucial for the specification of spatial data exchange formats and GIS interoperability standards.

Most previous efforts to analyze spatial relations have used relation calculus and have concentrated on spatial relations which are amenable to this treatment. The extension of relation calculus to a function calculus is discussed here, linking two previously unconnected tools. The two tools are not as different and their conceptual merging is in category theory (Barr and Wells 1990; Herring *et al.* 1990; Asperti and Longo 1991; Walters 1991). Function composition tables can be used similarly to relation composition tables; they show patterns which can then be succinctly formulated as rules.

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