Measuring Semantic Similarity between Geospatial Conceptual Regions

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Abstract. Determining the grade of semantic similarity between geospatial concepts is the basis for evaluating semantic interoperability of geographic information services and their users. Geometrical models, such as conceptual spaces, offer one way of representing geospatial concepts, which are modelled as n-dimensional regions. Previous approaches have suggested to measure semantic similarity between concepts based on their approximation by single points. This paper presents a way to measure semantic similarity between conceptual regions—leading to more accurate results. In addition, it allows for asymmetries by measuring directed similarities. Examples from the geospatial domain illustrate the similarity measure and demonstrate its plausibility.

1 Introduction

Semantic similarity measurements between concepts are the basis for establishing semantic interoperability of information services. To ensure successful communication between *geographic* information services and their users, it needs to be determined how similar their used *geospatial* concepts are. There exist various approaches to measure such similarity between concepts, depending on the concepts' types of representation. A common approach to representing concepts is based on geometrical models, where concepts are modelled as n-dimensional regions. Semantic similarity between concepts has previously been determined by approximating the regions through points and then measuring the distances between them. Such approximation inevitably leads to a loss of information and is therefore an inaccurate measure of similarity between concepts. In this paper, we present an approach of measuring semantic similarity between conceptual regions instead of their pointwise estimates. Such method improves the quality of the measurements by enhancing the accuracy of its results.

For the formal representation of conceptual regions we utilize Gärdenfors' idea of conceptual spaces—sets of quality dimensions within a geometrical structure [1]. Concepts can then be represented as n-dimensional regions in a vector space. The measurement of semantic similarity between conceptual regions is based on applying previously defined distance measures, such as given by instances of the Minkowski metric, to vectors forming the convex boundaries of the concepts whose similarities get evaluated. This way, a semantic distance function between two conceptual regions can be established. Furthermore, with the presented approach it is possible to account for asymmetries of similarity judgements, i.e., concepts are judged to be more similar to their superconcepts than vice versa. Examples from the geospatial domain are used throughout the paper to illustrate the semantic similarity calculations and their interpretations: we represent concepts such as mountain, hill, and lowland in a conceptual space according to their shapes through the dimensions 'height' and 'width'.

Section 2 introduces formal conceptual spaces and gives an overview of geometrical similarity measures. Section 3 describes the semantic similarity measure for conceptual regions and thereby applies the Euclidean distance function for calculation of similarity values. In section 4 we demonstrate how the proposed measure accounts for the fact that similarity judgements may be asymmetric. Section 5 focuses on the illustration and interpretation of distance values in different topological configurations (e.g., meeting and overlapping) of conceptual regions. The final section provides conclusions and directions for future work.

2 Related Work

This section defines formal conceptual spaces and introduces geometrical similarity measures.

2.1 Formal Conceptual Spaces

The idea of a conceptual space was introduced by Peter Gärdenfors as a framework for representing information at the conceptual level [1]. Such representation rests on the foundation of cognitive semantics [2], asserting that meanings are mental entities—mappings from expressions to conceptual structures, which themselves refer to the real world. Conceptual spaces can be utilized for knowledge representation and sharing, and support the paradigm that concepts are dynamical systems [3]. According to Gärdenfors, a conceptual space is a set of quality dimensions with a geometrical or topological structure for one or more domains. A domain is represented through a set of integral dimensions, which are distinguishable from all other dimensions. For example, the colour domain is formed through the dimensions hue, saturation, and brightness. Concepts cover multiple domains and are modelled as n-dimensional regions. Every object or member of the corresponding category is represented as a point in the conceptual space. This allows for expressing the similarity between two objects as the distance between their points in the space. Recent work by Gärdenfors deals with the idea of representing actions and functional properties in conceptual spaces [4].

In [5], a methodology to formalize conceptual spaces as vector spaces is presented. Formally, a conceptual vector space is defined as $\mathbf{C}^n = \{(c_1, c_2, \ldots, c_n) \mid c_i \in \mathbf{C}\}$ where the c_i are the quality dimensions. A quality dimension can also represent a whole domain and in this case $c_j = \mathbf{D}^n = \{(d_1, d_2, \ldots, d_n) \mid d_k \in \mathbf{D}\}$. The fact that vector spaces have a metric allows for the calculation of distances between points in the space. This can also be utilized for measuring distances between concepts, although it requires their approximation by "prototypical points." In order to calculate these so-called *semantic distances* between instances of concepts all quality dimensions of the space must be represented in the same relative unit of measurement. Assuming a normal distribution, this is ensured by calculating the z scores for these values, also called z-transformation [6]. For specifying different contexts one can assign weights to the quality dimensions of a conceptual vector space. This is essential for the representation of concepts as dynamical systems. In this case $\mathbf{C}^n$ is defined as $\{(w_1c_1, w_2c_2, \ldots, w_nc_n) \mid c_i \in \mathbf{C}, w_j \in \mathbf{W}\}$ where $\mathbf{W}$ is the set of real numbers.

2.2 Geometrical Similarity Measures

There exist a number of approaches to assess semantic nearness in a conceptual space with quite differing philosophies: some focus on angle or length difference, and others on the distance between vectors. Following Jones and Furnas [7] we choose a geometric representation with iso-similarity contours to demonstrate the semantic differences of the similarity functions: Moving an object along a contour line—analogous to contours in topographic maps—does not have an effect on its similarity value.

Euclidian- and City-block Distance Measure

The most common way of measuring similarity in conceptual spaces is the Minkowski metric (equation 1) which measures semantic distance in analogy to spatial distance. The Minkowski metric is a generic formula: r=1 results in the city-block distance and r=2 in the Euclidian distance. According to the city-block metric the distance equals the sum of the absolute distances of each dimension and the Euclidian distance is computed as the square root of the sum of the dimension-wise squared differences [8]. Similarity s is a linear decaying function of the semantic distance d [9, 10].

$$d(q,c) = \left[\sum_{i=1}^{n} |q_i - c_i|^r \right]^{1/r} \quad (\mathbf{1})$$

Figure 1 shows the iso-similarity contours for the Euclidian and city-block metrics: Vectors along one contour line all have the same similarity to the query vector. In the two-dimensional figures the Euclidian similarity contours are circular and the city-block contours are quadratic.

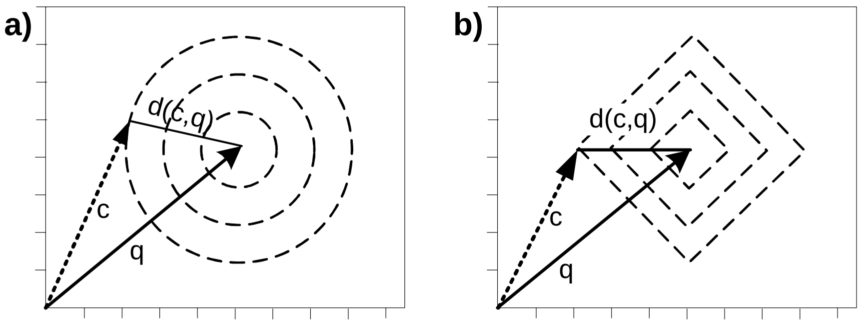

Fig. 1. Geometric comparison of the Euclidian (a) and the city-block distance measure (b).

Johannesson and Gärdenfors demonstrated in experimental studies—subjects had to rate the similarity between different mollusc shells and beetles—the usability of the Minkowski metric—especially the different underlying assumptions when Euclidian and city-block metrics are applied—within conceptual spaces [1, 11-13].

Cosine Similarity Measure

The cosine measure (equation 2) is a normalized inner product of two vectors: The inner product is divided by the product of the Euclidian vector lengths.

$$s(q,c) = \frac{\sum_{i=1}^{n} q_i * c_i}{\sqrt{\sum_{k=1}^{n} (q_k)^2} * \sqrt{\sum_{j=1}^{n} (c_j)^2}} \qquad \textbf{(2)}$$

Because of the Euclidian length normalization, all vectors having the same direction are transformed into the same unit vector regardless of their length (figure 2). Therefore all vectors on the same radiating line—the iso-similar contour—have the same similarity. Only the angle of separation influences the similarity value: The greater the angle between two vectors the lower is their similarity. In the two-dimensional representation of figure 2 the iso-similarity contours are lines with symmetrical similarity values on both sides of the query vector, but in an n-dimensional space the contours are cone-shaped. The cosine similarity measure is bounded from zero to one [7]. It is used, for example, to determine similar terms in a concept space [14]. The pseudo-cosine measure shown on the right side of figure 2 is similar to the cosine measure but normalized by the city-block length of the vectors.

This set of similarity measures was chosen because they are most frequently used in cognitive spaces, but there exists a variety of other similarity measures for vector spaces such as the dice measure, overlap measure and the Jaccard measure [7]. Besides the mentioned similarity measures for vector spaces there exist a number of other approaches to measure semantic similarity such as the feature matching model [15], Matching-Distance Similarity Model [16, 17], Distance [18] and the transformational model [19]. However, these are based on different representational models. Here, we consider only geometric similarity measures based on conceptual spaces.

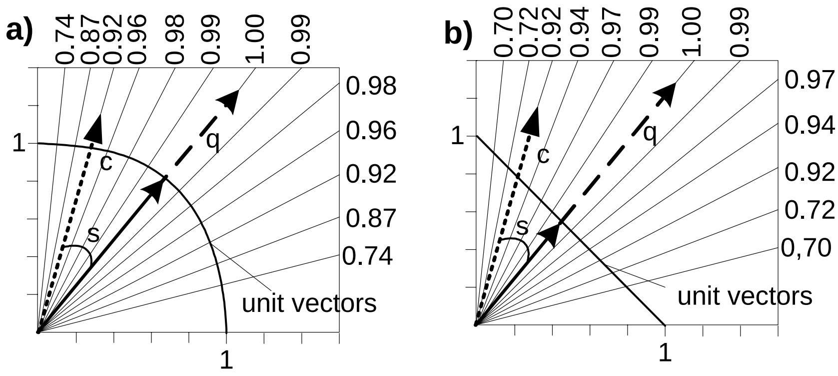

Fig. 2. Geometric comparison of the Cosine (a) and Pseudo-Cosine similarity measure (b) [7].

Previous distance measures for concepts in conceptual spaces first reduce concepts to a single point such as the balance point or centre point—often representing their prototypes [5, 20]. Then similarity measures for instances are applied [21]. By reducing concepts to single points or instances, the expressiveness as well as the significance of the distance measure are reduced. Neither a balance point nor the centre point can fully represent the semantics of a concept. The semantic similarity measure proposed in the following section overcomes this shortcoming.

3 Semantic Similarity between Geospatial Concepts

Current similarity measures confine themselves to estimating the similarity between instances, i.e. between points[1] in the vector space. The semantic similarity measure between geospatial concepts presented here is based on the similarity measure of instances, but measures the distance between concepts represented as convex regions in space. Similarity gets calculated in a two-step process: At first the concepts are reduced to a set of vector pairs. This way we transform the convex regions of concepts into a format to which the original similarity measures can be applied. Then the similarity measure is used for this set of vector pairs to calculate the similarity value. After stating some preliminary assumptions, section 3.2 explains how to calculate the vector pairs between concepts and section 3.3 focuses on the calculation of the similarity value.

3.1 Preliminary Assumptions

Before describing the similarity measurement procedure we need to introduce some preliminary assumptions: A concept is modelled as a convex region in an n-dimensional space, i.e. the region is continuous, completely closed and the hull of

[1] A point is represented by a vector in a vector space. We use the term 'point' to underline the difference between instances modelled as points and concepts modelled as regions.

the region is convex. Extreme distance values occur for position vectors whose end-points lie on the hull of the region. Figure 4 illustrates that the distances measured from a vector in the inside of query concept 'hill' (vector q_i) lie between the maximum and minimum distance values measured somewhere on the hull of the region (here vectors q_3 and q_6). Therefore it is sufficient to consider only the hull of the region representing query concept 'hill' to estimate the distance values.

The considerations about the similarity measure start with the simplifying model that the convex hull consists of a set of discrete points. Later we transfer the findings to a continuous function. We further assume that all concepts have the same dimensions.

Like the similarity measures in section 2.2 the illustration is simplified by representing concepts in a two-dimensional space. In the text we explain the findings for n-dimensional spaces.

3.2 Calculating Vector Pairs between Concepts

The hull of query concept Q is formed by the endpoints of a set of position vectors $S_Q=\{q_1, q_2, \ldots, q_n\}$. The first step of the distance calculation between Q and a concept C aims at defining for each vector q_i in S_Q one or several corresponding vectors of C. All vectors form vector pairs with q_i and these are the basis for the similarity calculation. The identification which vector pairs reflect best human similarity measurement is an important question. Figure 3 illustrates three different strategies.

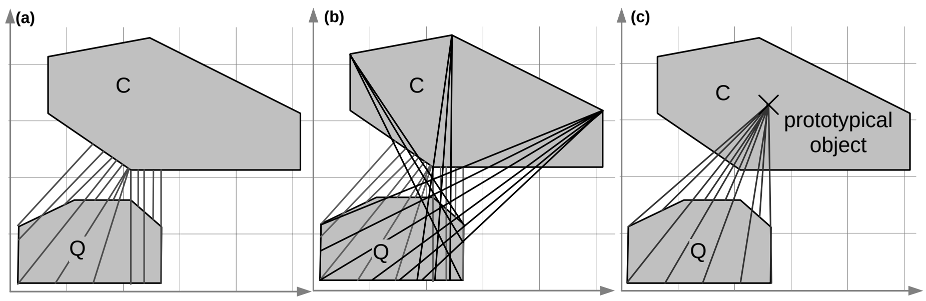

Fig. 3. Different strategies can be applied to identify for each vector q_i one or several corresponding vectors of a concept C: (a) searching for the vector with minimum distance, (b) searching for the vectors with either minimum or maximum distance, or (c) defining a prototypical object as reference object for the similarity calculations.

Strategy (a) is inspired by the idea that humans intuitively assess similarity by comparing a concept Q with that instance of the other concept C which is most similar to Q. Therefore the corresponding vector has the minimum distance to a vector q_i.

Strategy (b) supposes that not only the most similar, but also the most dissimilar instances of a concept influence human similarity assessment. Therefore there exist two corresponding vectors of C: one with minimum and another with maximum distance to a vector q_i.

Strategy (c) assumes that there exists a reference instance—e.g. a prototype of the concept—which is used as corresponding vector for all vectors q_i. In this case the similarity measure is only influenced by the shape of the conceptual region Q and concept C is reduced to one prototypical point.

Extensive human subject tests are required to substantiate the choice for one strategy. Such investigation is important, but lies outside the scope of this paper which focuses on developing a formal procedure for measuring similarity between conceptual regions.

$$nearVec(q_i) = \min_{\forall x \in C}\left(\left|f_c(x) - q_i\right|\right),\ q_i \in S_Q \qquad \textbf{(3)}$$

For the following calculations we apply strategy (a). Therefore we compute for each vector q_i in S_Q the vector c in C with the minimum distance to q_i according to the following formula (equation 3). Concept C is given by the function $f_c(x)$.

$$\left(q_i, nearVec(q_i)\right),\ q_i \in S_Q \qquad \textbf{(4)}$$

This vector pair (equation 4) consisting of vector q_i and the nearest vector in concept C *nearVec(q_i)* specifies the corresponding vectors between which the semantic distance is measured.

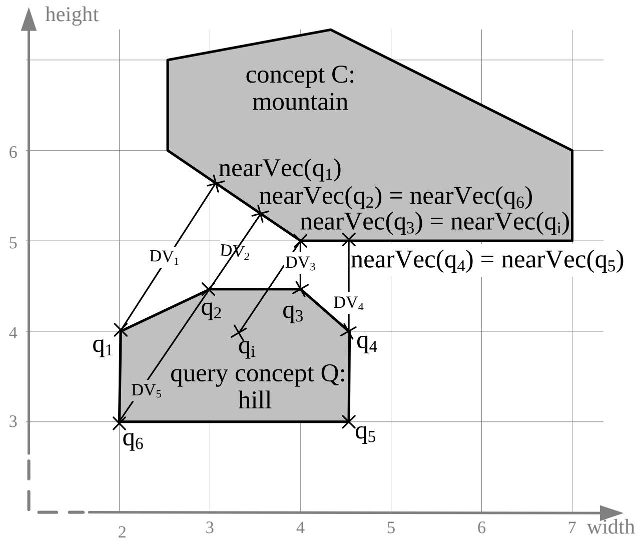

Fig. 4. Computation of corresponding vectors from query concept Q to concept C[2].

[2] For demonstration purposes we represent the concepts in the example by two dimensions only. For a complete description though, more dimensions such as 'shape' etc. are needed. The dimensions of the conceptual space 'width' and 'height' are measured in some standardized unit. For the calculation of semantic distances it is required to represent all dimensions in the same relative unit of measurement. The original, non-standardized units—in this example width and height can be measured in meters or kilometres—are standardized by the z-transformation [6].

Figure 4 illustrates the procedure: for each point q_i on the hull of query concept 'hill' one searches for the vector of concept 'mountain' with the minimum distance as illustrated by the difference vector DV_i. The vector pair determination depends also on the applied similarity measure: For cosine measures strategy (a) aims at minimizing angle size and therefore computes for each vector q_i in S_Q the position vector c of C with the smallest angle difference.

3.3 Applying Euclidian Distance Measure to Calculate Distance Value

All existing similarity measures explained in section 2 can be applied to the vector pairs introduced in section 3.2. This paper focuses on the most commonly used semantic similarity measure in conceptual spaces: the Euclidian distance measure.

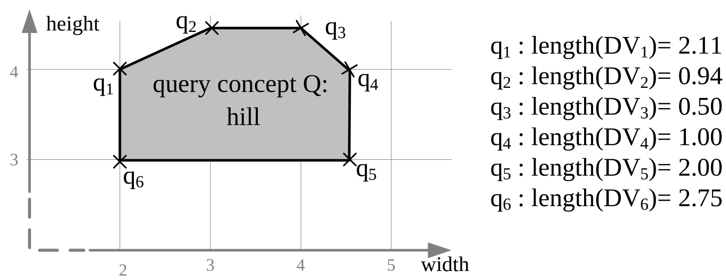

Fig. 5. Distances illustrated as values for each point q_i.

The Euclidian distance between two vectors in an n-dimensional space is measured by calculating the absolute difference vector DV_i. The length of vector DV_i is the semantic distance value for point q_i. Therefore each vector q_i of the hull of Q has a distance value to concept C. The hull of an n-dimensional region is an (n-1)-dimensional object in an n-dimensional space.

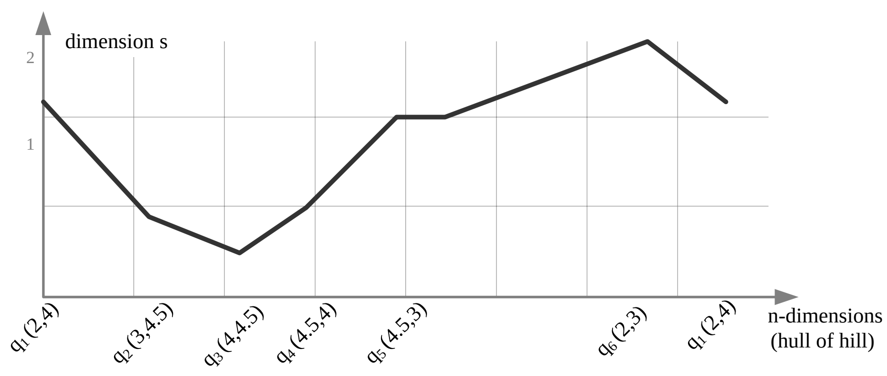

Fig. 6. Distances illustrated as distance function $DV(q_i)$.

The distance values can be represented on a dimension s. For better understanding we illustrate the distance values in two different ways: In figure 5 dimension s is represented by assigning the distance values to each point of the hull of query concept 'hill'. Figure 6 shows the dimension s as a function of the hull of concept 'hill'. Both figures illustrate the same fact using different representations.

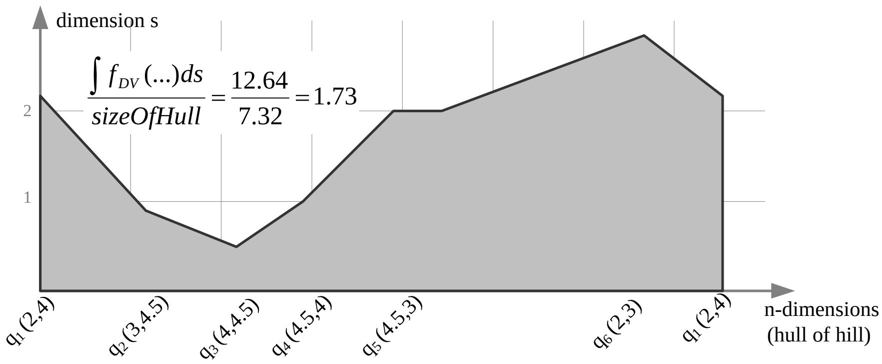

Fig. 7. Transformation of the pointwise Euclidian distance values into one distance value.

The Euclidian distance measure evaluates the length of the distance vectors. Distances near to zero indicate that concept 'hill' lies very close to concept 'mountain'. A small value on dimension s stands for high similarity. The minimum distance is for vector (4,4.5) and the maximum distance is for vector (2,3).

$$\frac{\int f_{DV}(...)ds}{sizeOfHull} \tag{5}$$

Currently with the finite set of vectors q_i the semantic distance can only be approximated. For a continuous computation we use the integral over the semantic distance function. Since the integral depends not only on the value of dimension s, but also on the size of the concept's hull—the bigger the hull the greater is the interval on the hull-dimensions—a normalization factor such as the size of the hull is required. Figure 7 shows the computation of the integral with respect to distance dimension s to estimate the distance value (equation 5). The semantic distance from concept 'hill' to concept 'mountain' is 1.73.

4 Asymmetric similarities

Psychologists found that the perceived similarity between two stimuli is not necessarily symmetric: non-prominent objects are more similar to a prominent object than vice versa [20]. In 1977, Amos Tversky [22-24] proposed a similarity measure allowing for asymmetric similarities. Geometric similarity measures are based on multidimensional spaces and assume metric properties such as minimality, symmetry and triangle inequality between items. The inability to deal with asymmetric similarities between objects and concepts is probably the most heavily criticized aspect of geometric similarity measures and was the reason for various extensions of conceptual spaces [25, 26].

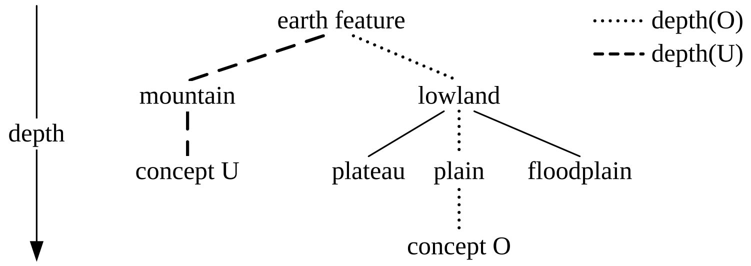

Fig. 8. Depth of concepts in the concept hierarchy (see also [16]).

In the geospatial domain, Rodriguez and Egenhofer developed the Matching-Distance Similarity Measure (MDSM) which accounts for asymmetries in similarity assessment [16, 17, 27, 28]: One component of the MDSM is Tversky's Feature Matching Model, which becomes an asymmetric measure depending on the choice of parameters α and β in the contrast model [22, 23].

$$\alpha(U,O)=\begin{cases}\dfrac{depth(U)}{depth(U)+depth(O)}, depth(U)\le depth(O)\\ 1-\dfrac{depth(U)}{depth(U)+depth(O)}, depth(U)>depth(O)\end{cases} \quad (6)$$

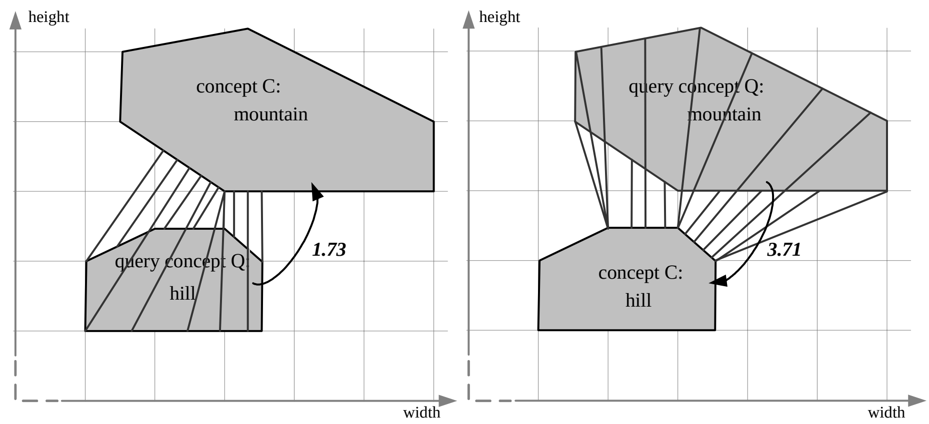

Fig. 9. Asymmetric semantic distance between two concepts.

Following the idea that people perceive similarity from a subconcept to its superconcept greater than the similarity from the superconcept to the subconcept, and that the superconcept is commonly used as a basis for the similarity judgment, Rodriguez and Egenhofer developed a formula (equation 6) to compute these parameters by building the ratio of the concepts' depth in the ontology (figure 8). In general, concepts deeper in the hierarchy are smaller—in terms of size of their represented conceptual regions—than their superconcepts, which have a greater degree of generalization.

The semantic distance measure proposed in this paper reflects the above described observation: concepts with a high degree of specialization covering a small region in the conceptual space tend to be more similar to general concepts than vice versa. Figure 9 shows how the similarity measure between concepts works and illustrates the effect of asymmetry. On the right hand side the query concept Q is much broader than the compared concept C. Therefore the semantic distance is greater and the similarity value is smaller than in the figure on the left side.

5 Illustration and Interpretation of the Distance Value

The following examples illustrate the results of the proposed semantic similarity measure for different topologic configurations and give an interpretation of the semantic distance values. Disjoint concepts were already discussed in section 3. Here we focus on meeting, overlapping, inside/containing and covering/covered by concepts. We refer to Egenhofer's definition of the topologic operators 'disjoint', 'meet', 'overlap', 'inside', 'contains', 'covers' and 'covered by' [29, 30].

5.1 Meeting and Overlapping Concepts

Figure 10 shows two meeting concepts 'steep face' and 'mountain' and their semantic distance function. For the interval where the conceptual regions 'steep face' and 'mountain' meet, the semantic distance is zero. The length of the interval is the same as the length of the contact. From this distance function it is not possible to distinguish whether 'steep face' meets 'mountain' from outside—the interiors of 'steep face' and 'mountain' do not intersect, but their boundaries do—or one covers the other, i.e. their interiors and boundaries intersect (see also section 5.2).

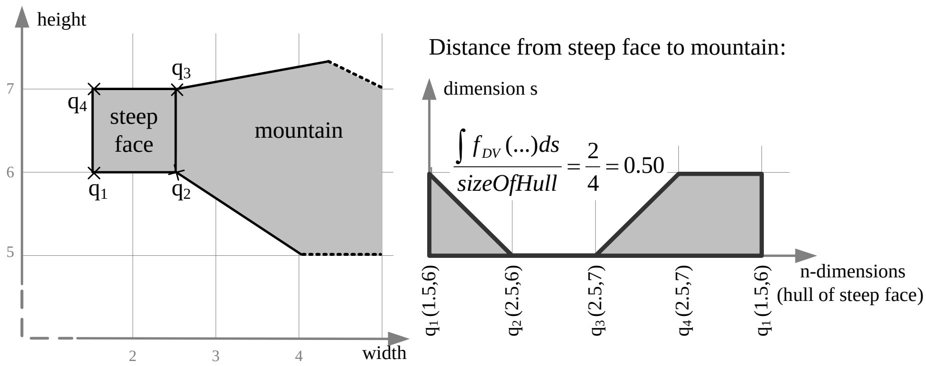

Fig. 10. Meeting concepts and their semantic distance function.

Figure 11 shows two overlapping concepts with their semantic distance function. The semantic distance between overlapping concepts is zero for the whole overlap. Therefore we cannot resolve the difference between meeting and overlapping concepts purely from the distance function. However, from the distance function one can gather information about the topology of both concepts:

1. If the distance function becomes zero, both concepts either meet (figure 10), overlap (figure 11), the query concept covers the other concept ('plateau' covers 'lowland' in figure 12), or it is inside another concept ('plain' is inside 'lowland').
2. If the distance function does not become zero at any time and every vector in C is also vector in Q, then concept Q contains concept C ('lowland' contains 'plain' in figure 12). If such a vector does not exist, then concepts Q and C are disjoint.

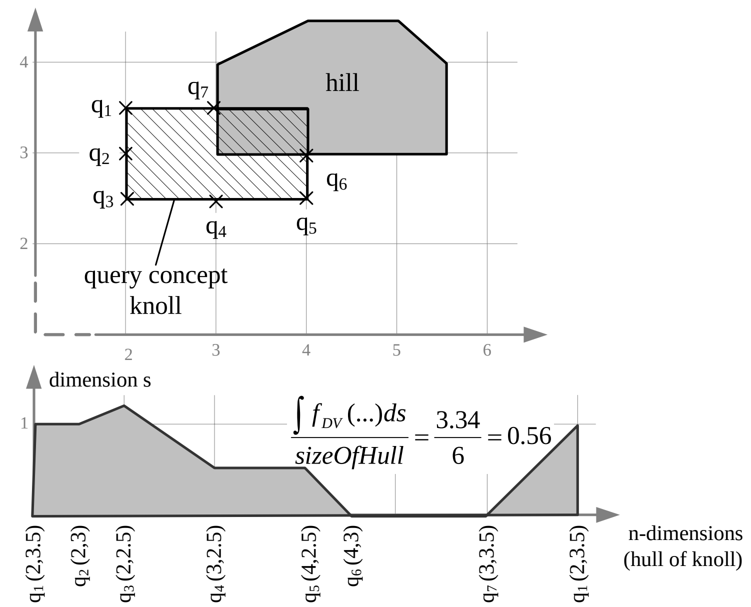

Fig. 11. Overlapping concepts and their semantic distance function.

To estimate the difference between meeting, overlapping, covering concepts, and concepts being inside other concepts, an additional measure computing the degree of overlap is required to refine the similarity values. The ratio of the overlapping and non-overlapping parts of the region is a good indicator for the degree of semantic overlap and therefore also for the similarity. The greater the overlap and the less the non-overlapping parts, the higher is the similarity between both concepts. A brute-force algorithm can be used for computing overlap of convex hulls: the plain SWEEP algorithm is also applicable in the 3-dimensional case (for detailed explanation see [31, 32]). Such computation is important, but outside the scope of this work.

5.2 Concepts Being Inside or Covered by Other Concepts

Figure 12 shows concepts inside or covering other concepts: 'plain' lies inside 'lowland', i.e. there is a complete overlap. Semantically interpreted this means that 'plain' is a subconcept of 'lowland'. If a concept is inside another concept, the distance values from the superconcept to the subconcept are always greater zero (for covering concepts the distance is greater or equal to zero). The distance values from the subconcepts 'plain' respectively 'plateau' to their superconcept 'lowland' are zero. The overlap measure can be used for further distinction.

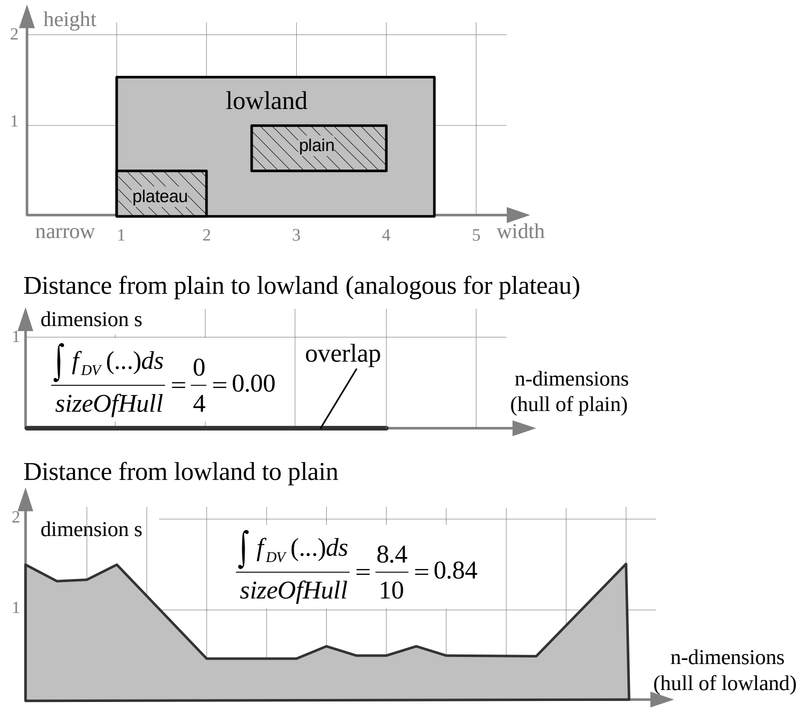

Fig. 12. Overlapping concepts with different topologic configurations.

Figure 13 gives an overview of the concepts discussed above in one conceptual space. The given semantic distance values are based on the described similarity measure and do not include additional refinements of an overlap measure. These semantic distances must be transformed into a similarity value according to a similarity function, for example a linear decaying function of these semantic distances.

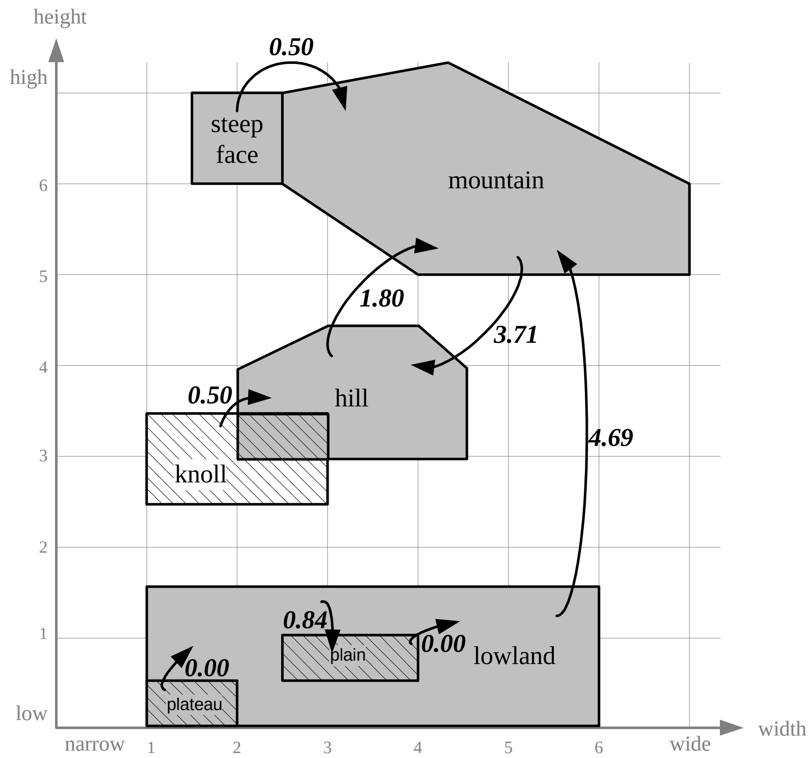

Fig. 13. Similarity values between concepts with different topologic relations in one conceptual space.

6 Summary and Future Work

This paper develops a geometric similarity measure between concepts modelled as conceptual regions in a conceptual space. Previous approaches reduce concepts to a prototypical point and use these as input for pointwise similarity measures. The similarity between concepts is therefore reduced to the similarity of their prototypes. Such reduction of regions to single points inevitably leads to a loss of information. These measures neither account for the shape of the conceptual regions, nor for their size. The similarity measure presented in this paper includes the whole conceptual region of a query concept for similarity calculation. Shape, size and distances of a concept to another concept influence the similarity measure. Moreover, this directed similarity measure accounts for the fact that people's similarity judgments are asymmetric.

The paper leads to different directions for future research:

1. Geospatial concepts are often complex with non-obvious dimensions. We simplified the concept description in the example by representing only two

dimensions. The underlying quality dimensions for a concept, its values on a dimension and the dependencies between dimensions can be identified by human subject tests (e.g. [13]). As well multi-dimensional scaling can be used to identify potential dimensions used by humans to judge similarity.

2. Here we make the simplifying assumption that both concepts are described by the same, independent dimensions. However, many concepts are represented by different numbers of dimensions. Future research needs to investigate whether it is feasible to either leave out different dimensions and consider only common ones, or whether missing dimensions have a negative impact on the similarity of concepts. Sometimes, different dimensions can be mapped to each other (see for example the mapping of RGB to HSB colours in [5]). Dependencies between dimensions may be discovered in human subject tests—e.g. [13]—which leads to non-orthogonal axes in the representation.
3. Since the determination of vector pairs is a unidirectional process—for each vector of the hull of Q the corresponding vector of C is determined—the size and shape of Q has a great influence on the similarity function. However, vectors in C that do not belong to a vector pair have no effect. When applying the Euclidian distance measure, the part of C being far away from Q does not influence the similarity at all. Future work must investigate empirically whether it is justifiable to consider only the part of a concept C with the minimum distance. Other strategies must be investigated (see section 3.2). We propose to include the distribution function of instances of the concept in the similarity measure, e.g. consider only that part of a concept with the density of instances larger than a given marginal value.
4. People's similarity judgments are highly dependent on their tasks and the general context. Future work needs to compare the calculated similarity values with results from human subject tests using different scenarios. Differences in similarity values for different contexts could be represented in conceptual spaces by assigning weights to the quality dimensions.

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